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Low–Energy QCD*

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Abstract

After a brief introduction to chiral perturbation theory, the effective field theory of the standard model at low energies, two recent applications are reviewed: elastic pion–pion scattering to two–loop accuracy and the complete renormalized pion–nucleon Lagrangian to $O(p^3)$ in the chiral expansion.

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1 THE STANDARD MODEL AT LOW ENERGIES

At low energies ($E \ll 1$ GeV), the simplicity of the QCD Lagrangian is deceptive. There are no "direct" signs of quarks and gluons in the confinement regime. Instead, the relevant degrees of freedom are first of all the hadrons that are stable under the strong interactions: the pseudoscalar mesons and the lowest–lying baryons. At a second stage, meson and baryon resonances can be included.

In principle, we are told to integrate out the fundamental degrees of freedom (quarks and gluons) to arrive at a field theory of the observed hadronic fields. In the confinement regime, this procedure is under theoretical control only for the chiral anomaly (Wess and Zumino, 1971). In practice therefore, one uses the symmetries of QCD and of the standard model in general to arrive at an effective field theory at low energies (Weinberg, 1979) called chiral perturbation theory (CHPT) (Gasser and Leutwyler, 1984, 1985; Leutwyler, 1994a). The crucial role in the construction of this effective field theory is played by the spontaneously broken chiral symmetry with the pseudoscalar mesons as corresponding (pseudo-) Goldstone bosons. Referring to recent reviews (Bernard et al., 1995; Ecker, 1995b; Leutwyler, 1994b; Pich, 1995; de Rafael, 1995) for a more extensive coverage of CHPT, I list here only a few salient features.

- As is the case for effective field theories with spontaneously broken symmetries in general, CHPT is a non–renormalizable quantum field theory.
- As required by unitarity, a consistent low–energy expansion entails a loop expansion. Since the loop amplitudes are in general divergent, the theory has to be regularized and renormalized.
- Once this has been achieved, CHPT incorporates all the usual properties of a bona fide quantum field theory, at least in the perturbative sense: unitarity, analyticity, crossing, ...
- The symmetries of the standard model are manifest by construction.
- There is no double–counting in CHPT: only hadronic fields, but neither quarks nor gluons appear in the chiral Lagrangian.
- All the short–distance structure is encoded in certain coupling constants, the so–called low–energy constants (LECs). In pure CHPT with only pseudoscalar mesons and low–lying baryons, even the resonances are included among the short–distance effects. More generally, the LECs describe the influence of all degrees of freedom not explicitly contained in the effective chiral Lagrangian.
- CHPT is not a special model for low–energy QCD like, e.g., the Nambu–Jona-Lasinio model (Nambu and Jona-Lasinio, 1961), but a quantum field theory framework to construct the most general solution of the Ward identities of QCD (the standard model in general).

There is a price to pay for this generality. Since the LECs parametrize the solutions of the Ward identities, they are by definition not constrained by the symmetries. Additional input is needed to make CHPT predictive, especially in higher orders of the chiral expansion. This information (for a recent review, see Ecker, 1995a) comes either from experiment or from additional theoretical input (resonance saturation, QCD sum rules, large–N_c expansion, lattice QCD, Nambu–Jona-Lasinio type models, skyrmions, ...).

2 EFFECTIVE CHIRAL LAGRANGIAN

In the real world, there is no chiral symmetry. Even in the limit of vanishing quark masses, all theoretical and phenomenological evidence indicates that the chiral group $G = SU(N_f)_L \times SU(N_f)_R$ (for N_f=2 or 2 massless quarks) is spontaneously broken to the diagonal (vectorial) subgroup $SU(N_f)_V$. There is a standard procedure (Coleman et al., 1969; Callan et al., 1969) how to realize a spontaneously broken symmetry on quantum fields. In the special case of chiral symmetry with its parity transformation, the Goldstone fields φ can be collected in a unitary matrix field $U(\varphi)$ transforming as

$$U(\varphi) \stackrel{G}{\rightarrow} g_R U(\varphi) g_L^{-1} \ , \qquad (g_L, g_R) \in G \tag{2.1}$$

under chiral transformations.

The chiral symmetry is in addition broken explicitly by non–vanishing quark masses and, if the weak interactions are included, through the handedness of the weak interactions. Restricting the attention first to the strong, electromagnetic and semileptonic weak interactions, the most convenient way to introduce the explicit chiral symmetry breaking is via the introduction of external scalar (s), pseudoscalar (p), vector (v) and axial–vector (a) fields (Gasser and Leutwyler, 1984, 1985) that contain both the quark masses and external photons and W bosons.

Although this framework is sufficient for the applications I am going to discuss in this talk, let me emphasize for completeness that the formalism must be extended if one wants to include "internal" photons and W bosons. Turning to the non–leptonic weak interactions, one first has to integrate out the W boson together with the heavy quarks to arrive at an effective Hamiltonian still at the fundamental quark level that transforms under chiral transformations as

$$\mathcal{H}_{\rm eff}^{\Delta S=1} \sim (8_L, 1_R) + (27_L, 1_R) \tag{2.2}$$

in the $\Delta S = 1$ sector. This effective Hamiltonian is then realized by an effective chiral Lagrangian at the level of mesons and baryons that must, of course, have the same transformation property (2.2).

The situation is again different for virtual photons relevant for the treatment of electromagnetic corrections. Virtual photons cannot be integrated out to produce a local Hamiltonian at the quark level. Instead, one has to introduce the photon as a dynamical field at the hadronic level. In addition, one must include the general chiral Lagrangian of $O(e^2)$ that transforms as the product of two electromagnetic currents. The mesonic Lagrangian of $O(e^2p^2)$ has only recently been constructed (Urech, 1995; Neufeld and Rupertsberger, 1995).

CHPT is based on a two–fold expansion. As a low–energy effective field theory, it is an expansion in small momenta. On the other hand, it is also an expansion in the quark masses m_q around the chiral limit. In full generality, the effective chiral Lagrangian is of the form

$$\mathcal{L}_{\rm eff} = \sum_{i,j} \mathcal{L}_{ij} \ , \qquad \mathcal{L}_{ij} = O(p^i m_q^j) \ . \tag{2.3}$$

The two expansions become related by expressing the pseudoscalar meson masses in terms of the quark masses. If the quark condensate is non–vanishing in the chiral limit, the squares of the meson masses start out linear in m_q [cf. Eq. (3.9)]. Assuming the linear terms to give the dominant contributions to the meson masses, one arrives at the standard chiral counting (Gasser and Leutwyler, 1984,1985) with $m_q = O(p^2)$ and

$$\mathcal{L}_{\rm eff} = \sum_{d} \mathcal{L}_{d} \ , \qquad \mathcal{L}_{d} = \sum_{i+2j=d} \mathcal{L}_{ij} \ . \tag{2.4}$$

Table 1: The effective chiral Lagrangian of the standard model

$\mathcal{L}_{\text{chiral dimension}}$ (# of LECs)	loop order
$\mathcal{L}_2(2) + \mathcal{L}_4^{\text{odd}}(0) + \mathcal{L}_2^{\Delta S=1}(2) + \mathcal{L}_0^{\gamma}(1)$	$L = 0$
$+ \mathcal{L}_1^{\pi N}(1) + \mathcal{L}_2^{\pi N}(7) + \ldots$	
$+ \underline{\mathcal{L}_4^{\text{even}}(10)} + \underline{\mathcal{L}_6^{\text{odd}}(32)} + \underline{\mathcal{L}_4^{\Delta S=1}(22, \text{octet})} + \underline{\mathcal{L}_2^{\gamma}(14)}$	$L = 1$
$+ \underline{\mathcal{L}_3^{\pi N}(24)} + \mathcal{L}_4^{\pi N}(?) + \ldots$	
$+ \mathcal{L}_6^{\text{even}}(111) + \ldots$	$L = 2$

An alternative way to organize the chiral expansion accounts for the possibility that the quark condensate might be much smaller than usually assumed. In that case, the leading–order contributions according to the usual counting would not necessarily give the dominant contributions to the meson masses. Although there are several arguments in favour of the standard counting, among them the validity of the Gell-Mann–Okubo mass formula for the pseudoscalar meson masses, the alternative approach of Generalized CHPT (see Knecht and Stern, 1995a for a recent review) is still a logical possibility. In the generalized picture, more terms appear at a given order that are relegated to higher orders in the standard counting. Obviously, this procedure increases the number of LECs at any given order. It should be kept in mind, however, that the effective chiral Lagrangian of the standard model is the same in the standard and in the generalized approach. In this talk, I will strictly adhere to the standard procedure, but I will briefly come back to Generalized CHPT in the discussion of $\pi\pi$ scattering.

The effective chiral Lagrangian of the standard model is shown schematically in Table 1. The subscripts of the different parts of this Lagrangian denote the chiral dimension according to the standard counting and the numbers in brackets indicate the appropriate number of LECs. The notation even/odd refers to the mesonic Lagrangians without/with an ε tensor (even/odd intrinsic parity). I have grouped together those pieces of the Lagrangian that have the same chiral order as a corresponding loop amplitude ($L = 0,\ 1,\ 2$). The Lagrangians $\mathcal{L}_n^{\Delta S=1}$ and $\mathcal{L}_n^{\gamma}$ describe non–leptonic weak interactions and virtual photons, respectively, but I have only included the purely mesonic parts. In fact, in the meson–baryon sector only the pion–nucleon Lagrangian is included, i.e. $N_f = 2$. On the other hand, the number of LECs in the purely mesonic Lagrangians are given for $N_f = 3$. As already emphasized in the beginning, the theory has to be renormalized once one reaches a chiral order where loop diagrams must be included. The parts of the effective chiral Lagrangian that have been completely renormalized are underlined in Table 1.

The Table shows the dramatic increase of the number of LECs with the chiral order. It is clear that one will never be able to measure all 111 LECs (Fearing and Scherer, 1994) in the strong meson Lagrangian of $O(p^6)$. As I will try to demonstrate, one can make meaningful predictions to $O(p^6)$ nevertheless.

3 ELASTIC PION–PION SCATTERING

Pion–pion scattering is a fundamental process for testing CHPT that involves only the pseudo–Goldstone bosons of chiral $SU(2)$. In the limit of isospin conservation ($m_u = m_d$), the scattering amplitude for

$$\pi^a(p_a) + \pi^b(p_b) \to \pi^c(p_c) + \pi^d(p_d) \tag{3.1}$$

is determined by a single scalar function $A(s,t,u)$ defined by the isospin decomposition

$$\begin{aligned} T_{ab,cd} &= \delta_{ab}\delta_{cd}A(s,t,u) + \delta_{ac}\delta_{bd}A(t,s,u) + \delta_{ad}\delta_{bc}A(u,t,s) \\ A(s,t,u) &= A(s,u,t) \end{aligned} \tag{3.2}$$

in terms of the usual Mandelstam variables s,t,u. The amplitudes $T^I(s,t)$ of definite isospin ($I = 0,1,2$) in the s–channel are decomposed into partial waves (θ is the center–of–mass scattering angle):

$$T^I(s,t) = 32\pi \sum_{l=0}^{\infty}(2l+1)P_l(\cos\theta)t_l^I(s) \ . \tag{3.3}$$

Unitarity implies that in the elastic region $4M_\pi^2 \le s \le 16M_\pi^2$ the partial–wave amplitudes t_l^I can be described by real phase shifts δ_l^I. The behaviour of the partial waves near threshold is of the form

$$\Re e \ t_l^I(s) = q^{2l}\{a_l^I + q^2 b_l^I + O(q^4)\} \ , \tag{3.4}$$

with q the center–of–mass momentum. The quantities a_l^I and b_l^I are called scattering lengths and slope parameters, respectively.

At lowest order in the chiral expansion, $O(p^2)$, the scattering amplitude is given by the current algebra result (Weinberg, 1966)

$$A_2(s,t,u) = \frac{s - M_\pi^2}{F_\pi^2} \ , \tag{3.5}$$

leading in particular to an $I = 0$ S–wave scattering length $a_0^0 = 0.16$. Near threshold, the chiral expansion for $SU(2)$ is expected to converge rapidly because the natural expansion parameter is of the order

$$\frac{4M_\pi^2}{16\pi^2 F_\pi^2} = 0.06 \ . \tag{3.6}$$

However, CHPT produces also singularities in the quark mass expansion (the so–called chiral logarithms) that may enhance this value. For an L–loop amplitude, the chiral logarithms appear in a general amplitude as $(\ln \frac{p^2}{\mu^2})^n$ with $n \le L$. Here, p is a generic momentum and the dependence on the arbitrary scale μ is compensated by the scale dependence of the appropriate LECs in the amplitude.

The scattering amplitude of $O(p^4)$ (Gasser and Leutwyler, 1983, 1984) has the general structure

$$\begin{aligned} F_\pi^4 A_4(s,t,u) = \ & c_1 M_\pi^4 + c_2 M_\pi^2 s + c_3 s^2 + c_4(t-u)^2 \\ & + F_1(s) + G_1(s,t) + G_1(s,u) \ . \end{aligned} \tag{3.7}$$

The functions F_1, G_1 are one–loop functions and the coefficients $c_1, \ldots, c_4$ of the general crossing symmetric polynomial of $O(p^4)$ are given in terms of the appropriate LECs (Gasser and Leutwyler, 1983, 1984) $l_i^r(\mu)$ ($i = 1,\ldots,4$) and of $\ln M_\pi^2/\mu^2$. Since four different LECs appear at this order, it is not surprising that chiral symmetry does not put any further constraints on the c_i. With

the phenomenological values of the LECs $l_i^r(\mu)$, Gasser and Leutwyler (1983, 1984) obtained $a_0^0 = 0.20 \pm 0.01$ to be compared with the value $a_0^0 = 0.26 \pm 0.05$ extracted from experiment (Nagels et al., 1979). One observes first of all quite a sizeable correction of $O(p^4)$ which can be attributed to a large extent to chiral logs and, secondly, a still bigger experimental value, albeit with a large error.

The value of a_0^0 has some bearing on the mechanism of spontaneous chiral symmetry breaking. If the pseudoscalar meson masses are not dominated by the lowest–order contributions proportional to the quark condensate [cf. Eq. (3.9)], the standard quark mass ratios could be significantly modified. This is precisely the option that Generalized CHPT proposes to keep in mind. In the generalized approach, the scattering lengths can not be absolutely predicted at $O(p^2)$, but they depend in addition on the quark mass ratio $r = 2m_s/(m_u + m_d)$ (Stern et al., 1993; Knecht et al., 1995b). Taking the experimental mean value for a_0^0 at face value would decrease the quark mass ratio r from its generally accepted value 26 to about 10.

After a period of rather little activity on the experimental side, there are now good prospects for more precise data on $\pi\pi$ scattering in the near future. Most promising are the forthcoming data from K_{e4} decays at the Φ–factory DAΦNE in Frascati which are expected to reduce the experimental uncertainty of a_0^0 to some 5% in $10^7 s$ running time (Baillargeon and Franzini, 1995). These experimental prospects have prompted two groups to attack the scattering amplitude of $O(p^6)$ (Knecht et al., 1995c; Bijnens et al., 1995). Knecht et al. have used dispersion relations to calculate the analytically non–trivial part of the amplitude and they have fixed some of the subtraction constants using $\pi\pi$ scattering data at higher energies. This approach does not yield the chiral logs which, although contributing only to the polynomial part of the amplitude, make an important numerical contribution (Colangelo, 1995; Bijnens et al., 1995). The standard field theory calculation of CHPT produces all those terms, but one has to calculate quite a few diagrams with $L = 0, 1$ and 2 loops to get the final amplitude. The two groups completely agree on the absorptive parts which are contained in the functions F_2, G_2 in the general decomposition

$$\begin{aligned} F_\pi^6 A_6(s,t,u) = \quad & d_1 M_\pi^6 + d_2 M_\pi^4 s + d_3 M_\pi^2 s^2 + d_4 M_\pi^2 (t-u)^2 + d_5 s^3 + d_6 s (t-u)^2 \\ & + F_2(s) + G_2(s,t) + G_2(s,u) \ . \end{aligned} \tag{3.8}$$

The coefficients $d_1, \ldots, d_6$ in the general crossing symmetric polynomial of $O(p^6)$ depend on the LECs $l_i^r(\mu)$ $(i = 1, \ldots, 4)$ of $O(p^4)$, on the chiral logs $(\ln M_\pi^2/\mu^2)^n$ $(n = 1, 2)$ and on six combinations of the LECs of $O(p^6)$ (Fearing and Scherer, 1994). Again, chiral symmetry does not constrain these six combinations. However, both chiral dimensional analysis and saturation by resonance exchange, which is known to work very well at $O(p^4)$ (Ecker et al., 1989a, 1989b), suggest values for these LECs that do not affect the threshold parameters in a dramatic way, especially not for the S–waves. The coefficients d_i are dominated on the one hand by the LECs $l_i^r(\mu)$ of $O(p^4)$ and by the chiral logs. Instead of ascribing a theoretical error to a_0^0 (which is dominated by the errors of the $l_i^r(\mu)$), I compare the predicted values to $O(p^n)$ for $n = 2, 4$ and 6 (Bijnens et al., 1995) to three significant digits (using the "old" value $F_\pi = 93.2$ MeV and the charged pion mass) :

n	2	4	6
a_0^0	0.156	0.201	0.217

The main conclusion concerning a_0^0 is that the correction of $O(p^6)$ is reasonably small and under theoretical control making a value as high as 0.26 practically impossible to accommodate within standard CHPT. Therefore, the forthcoming $\pi\pi$ data can be expected to either corroborate this prediction with significant precision or to shed serious doubts on the assumed mechanism of spontaneous chiral symmetry breaking through the quark condensate.

For a discussion of other threshold parameters and of the phase shifts themselves, I refer to Bijnens et al. (1995) for the standard CHPT calculation and to Knecht et al. (1995c) for the dispersion theory analysis.

At the level of precision we have reached with the $O(p^6)$ calculation, one may wonder about the size of electromagnetic and isospin violating corrections. Neglecting the tiny $\pi^0 - \eta$ mixing angle, the charged and neutral pion masses are equal to $O(p^2)$ without assuming $m_u = m_d$:

$$M^2_{\pi^+} = M^2_{\pi^0} = (m_u + m_d)B \ , \tag{3.9}$$

where B is proportional to the quark condensate. The mass difference between charged and neutral pions is almost entirely an effect of $O(e^2p^0)$ and it is determined by the Lagrangian $\mathcal{L}^\gamma_0(1)$ in Table 1. This effect is obviously non–negligible also for $\pi\pi$ scattering as can be seen, for instance, from the lowest–order expression for the $I = 0$ S–wave scattering length evaluated with either the charged or the neutral pion mass:

$$a^0_0 = \frac{7M^2_\pi}{32\pi F^2_\pi} = \left\{ \begin{array}{lll} 0.156 & \text{with} & M^2_{\pi^+} \\ 0.146 & \text{with} & M^2_{\pi^0} \end{array} \right. \ . \tag{3.10}$$

Clearly, the difference is comparable to the chiral correction of $O(p^6)$. The pion decay constant is also affected by radiative corrections, which have been estimated (Holstein, 1990; Review of Particle Properties, 1994) to move F_π down from 93.2 to 92.4 MeV. This decrease of F_π increases the $O(p^6)$ prediction for a^0_0 from 0.217 to 0.222 (Bijnens et al., 1995).

The question is then what other effects of $O(e^2p^0)$ appear in the $\pi\pi$ scattering amplitude. To answer this question, let us restrict the Lagrangian $\mathcal{L}^\gamma_0(1)$ to $N_f = 2$ and expand it in pion fields, using for convenience the so–called σ–parametrization for U:

$$\mathcal{L}^\gamma_0(1)(\text{pions}) = e^2 C\langle QUQU^\dagger\rangle(\text{pions}) = -\frac{2e^2C}{F^2}\pi^+\pi^- \ , \tag{3.11}$$

where Q is the quark charge matrix, C is the unique LEC of $O(e^2p^0)$ and $< \ldots >$ stands for the trace in two–dimensional flavour space. The conclusion is that there are no terms of $O(\pi^n)$ for $n > 2$. In other words, the Lagrangian $\mathcal{L}^\gamma_0(1)$ contributes only to the $\pi^+ - \pi^0$ mass difference, but not to the scattering amplitude itself. To leading $O(e^2p^0)$ therefore, electromagnetic corrections appear only in the kinematics and can easily be accounted for. The leading non–trivial electromagnetic effects for $\pi\pi$ scattering occur at $O(e^2p^2)$ and they are under investigation (J. Gasser, private communication). They can be expected to be quite a bit smaller than suggested by Eq. (3.10).

4 PION–NUCLEON LAGRANGIAN TO $O(p^3)$

We are looking for a systematic low–energy expansion of the pion–nucleon Lagrangian for single–nucleon processes, i.e. for processes of the type $\pi N \to \pi \ldots \pi N$, $\gamma N \to \pi \ldots \pi N$, $\gamma^* N \to \pi \ldots \pi N$ (including nucleon form factors), $W^* N \to \pi \ldots \pi N$. There is an obvious problem with chiral counting here: in contrast to pseudoscalar mesons, the nucleon four–momenta can never be "soft" because the nucleon mass does not vanish in the chiral limit. Although the problem can be handled at the Lagrangian level (Gasser et al., 1988; Krause, 1990), it reappears once one goes beyond tree level. The loop expansion and the derivative expansion do not coincide like in the meson sector. The culprit is again the nucleon mass that enters loop amplitudes through the nucleon propagators. In

the original relativistic formulation (Gasser et al., 1988), amplitudes of a given chiral order receive contributions from any number of loops.

A comparison between the nucleon mass and the chiral expansion scale $4\pi F_\pi$ suggests a simultaneous expansion in

$$\frac{\vec{p}}{4\pi F} \qquad \text{and} \qquad \frac{\vec{p}}{m}$$

where $\vec{p}$ is a small three–momentum. On the other hand, there is a crucial difference between $F \simeq F_\pi$ and $m \simeq m_N$: whereas F appears only in vertices, the nucleon mass enters a generic diagram via the nucleon propagator. The idea of Heavy Baryon CHPT (HBCHPT) (Jenkins and Manohar, 1991, 1992) is precisely to transfer m from the propagators to some vertices. The method can be interpreted as a clever choice of fermionic variables (Mannel et al., 1992) in the generating functional of Green functions (Gasser et al., 1988)

$$e^{iZ[j,\eta,\bar{\eta}]} = \int [dud\Psi d\bar{\Psi}] \exp[i\{S_M + S_{\pi N} + \int d^4x(\bar{\eta}\Psi + \bar{\Psi}\eta)\}] \ . \tag{4.1}$$

Here, $S_M + S_{\pi N}$ is the combined pion–nucleon action in the relativistic framework, Ψ is the nucleon field, η is a fermionic source and j stands for the previously introduced external fields (v, a, s, p).

In terms of velocity dependent fields N_v, H_v defined as (Georgi, 1990)

$$\begin{aligned} N_v(x) &= \exp[imv \cdot x] P_v^+ \Psi(x) \\ H_v(x) &= \exp[imv \cdot x] P_v^- \Psi(x) \\ P_v^\pm &= \frac{1}{2}(1 \pm \not{v}) \ , \qquad v^2 = 1 \ , \end{aligned} \tag{4.2}$$

with a time–like unit four–vector v, the pion–nucleon action $S_{\pi N}$ takes the form

$$\begin{aligned} S_{\pi N} &= \int d^4x \{\bar{N}_v A N_v + \bar{H}_v B N_v + \bar{N}_v \gamma^0 B^\dagger \gamma^0 H_v - \bar{H}_v C H_v\} \\ I &= I_{(1)} + I_{(2)} + I_{(3)} + \dots \ , \qquad I = A, B, C \ . \end{aligned} \tag{4.3}$$

The operators $A_{(n)}$, $B_{(n)}$, $C_{(n)}$ are the corresponding projections of the relativistic pion–nucleon Lagrangians $\mathcal{L}_{\pi N}^{(n)}$. Rewriting also the source term in (4.1) in terms of N_v, H_v, one can integrate out the "heavy" components H_v to obtain a non–local action in the fields N_v (Bernard et al., 1992). Expanding this non–local action in a power series in $1/m$, one obtains a Lorentz–covariant chiral expansion for the Lagrangian

$$\widehat{\mathcal{L}}_{\pi N}(N_v; v) = \widehat{\mathcal{L}}_{\pi N}^{(1)} + \widehat{\mathcal{L}}_{\pi N}^{(2)} + \widehat{\mathcal{L}}_{\pi N}^{(3)} + \dots \ , \tag{4.4}$$

which depends of course on the arbitrary four–vector v. Specializing to the nucleon rest frame (either in the initial or in the final state) with $v = (1, 0, 0, 0)$, (4.4) amounts to a non–relativistic expansion for the πN Lagrangian. In this Lagrangian, the nucleon mass m appears only in vertices, but not in the propagator of the transformed nucleon field N_v.

A given Lorentz covariant Lagrangian for the field N_v will in general not be Lorentz invariant because it depends on the arbitrary four–vector v. To guarantee Lorentz invariance, two procedures are possible. One can either impose reparametrization invariance (Luke and Manohar, 1992) on the Lagrangian (4.4) a posteriori or one can start directly from the fully relativistic Lagrangian which is Lorentz invariant by construction. The second approach has advantages especially in higher orders of the chiral expansion and it implies of course reparametrization invariance. This is the approach I am going to follow here.

The relativistic pion–nucleon Lagrangian of lowest order (Gasser et al., 1988),

$$\mathcal{L}_{\pi N}^{(1)} = \bar{\Psi}\left(i \not{\nabla} - m + \frac{g_A}{2} \not{u}\gamma_5\right)\Psi, \tag{4.5}$$

leads directly to the corresponding "non–relativistic" Lagrangian of $O(p)$:

$$\widehat{\mathcal{L}}_{\pi N}^{(1)} = \bar{N}_v(iv\cdot\nabla + g_A S\cdot u)N_v \ . \tag{4.6}$$

Here, m and g_A are the nucleon mass and the axial–vector coupling constant in the chiral limit, ∇ is a covariant derivative that includes in particular the photon field, u^μ is the vielbein field related to the matrix field U and $S^\mu = i\gamma_5\sigma^{\mu\nu}v_\nu/2$ is the spin matrix, the only remnant of Dirac matrices in the Lagrangian $\widehat{\mathcal{L}}_{\pi N}$.

At the next chiral order, $O(p^2)$, the Lagrangian $\widehat{\mathcal{L}}_{\pi N}^{(2)}$ consists of two pieces (Bernard et al., 1992). There is first a piece that is due to the expansion in $1/m$ with completely determined coefficients and there is in addition the non–relativistic reduction of the relativistic Lagrangian $\mathcal{L}_{\pi N}^{(2)}$. After a suitable field transformation of the nucleon field N_v, the Lagrangian assumes its most compact form (Ecker and Mojžiš, 1995c)

$$\begin{aligned}
\widehat{\mathcal{L}}_{\pi N}^{(2)} \ &= \ \bar{N}_v\left(-\frac{1}{2m}(\nabla\cdot\nabla + ig_A\{S\cdot\nabla, v\cdot u\})\right. \\
&\quad + \frac{a_1}{m}\langle u\cdot u\rangle + \frac{a_2}{m}\langle (v\cdot u)^2\rangle + \frac{a_3}{m}\langle\chi_+\rangle + \frac{a_4}{m}\left(\chi_+ - \frac{1}{2}\langle\chi_+\rangle\right) \\
&\quad \left. + \frac{1}{m}\varepsilon^{\mu\nu\rho\sigma}v_\rho S_\sigma[ia_5 u_\mu u_\nu + a_6 f_{+\mu\nu} + a_7 v_{\mu\nu}^{(s)}]\right) N_v \ ,
\end{aligned} \tag{4.7}$$

where χ_+ contains the quark mass matrix and the tensor fields $f_{+\mu\nu}$, $v_{\mu\nu}^{(s)}$ are the isovector and isoscalar parts of the external gauge fields including the electromagnetic field. The LECs a_i ($i = 1,\ldots,7$) are dimensionless and expected to be of $O(1)$ according to naive chiral dimensional analysis. They are in fact all known phenomenologically and I refer to Bernard et al. (1995) for an up-to-date review. For future purposes, let me single out two of them that are related to the nucleon magnetic moments in the chiral limit:

$$\begin{aligned}
a_6 \ &= \ \frac{\mu_v}{4} = \frac{1}{4}(\mu_p - \mu_n) \\
a_7 \ &= \ \frac{\mu_s}{2} = \frac{1}{2}(\mu_p + \mu_n) \ .
\end{aligned} \tag{4.8}$$

The first two terms in the Lagrangian (4.7) illustrate the difference between Lorentz covariance and invariance. The latter fixes the coefficients uniquely although covariance alone would seem to allow arbitrary coefficients. That these coefficients cannot be arbitrary becomes obvious when one realizes that the first term governs the Thomson limit for nucleon Compton scattering and the second one is responsible for the $O(p^2)$ contribution for pion photoproduction on nucleons at threshold. Of course, both amplitudes are completely determined by the nucleon charge and by g_A.

Chiral power counting (Weinberg, 1990) for the chiral dimension D of a generic CHPT diagram with N_d^M mesonic vertices and N_d^{MB} pion–nucleon vertices of $O(p^d)$,

$$D = 2L + 1 + \sum_d (d-2)N_d^M + \sum_d (d-1)N_d^{MB} \geq 2L + 1 \ , \tag{4.9}$$

tells us that loop diagrams enter at $O(p^3)$. As is to be expected in a non–renormalizable quantum field theory, those loop diagrams are in general divergent and must be regularized. Consequently,

the theory has to be renormalized to give results independent of the regularization procedure. The structure of the divergences (the divergence functional) can be calculated in closed form (Ecker, 1994) leading to scale–dependent LECs of $O(p^3)$. After applying again suitable field transformations, the complete πN Lagrangian of $O(p^3)$ is found to be (Ecker and Mojžiš, 1995c)

$$
\begin{aligned}
\widehat{\mathcal{L}}^{(3)}_{\pi N} \;=\; & \bar{N}_v \left(\frac{g_A}{8m^2}[\nabla_\mu,[\nabla^\mu, S\cdot u]] + \frac{1}{2m^2}\left[\left\{ i\left(a_5 - \frac{1-3g_A^2}{8}\right) u_\mu u_\nu \right.\right.\right. \\
& + \left.\left(a_6 - \frac{1}{8}\right) f_{+\mu\nu} + \left(a_7 - \frac{1}{4}\right) v^{(s)}_{\mu\nu}\right\} \varepsilon^{\mu\nu\rho\sigma} S_\sigma i \nabla_\rho + \frac{g_A}{2} S\cdot\nabla u\cdot\nabla \\
& - \left.\frac{g_A^2}{8}\{v\cdot u, u_\mu\}\varepsilon^{\mu\nu\rho\sigma} v_\rho S_\sigma \nabla_\nu - \frac{i g_A}{16}\varepsilon^{\mu\nu\rho\sigma} f_{-\mu\nu} v_\rho \nabla_\sigma + \text{h.c.}\right] \\
& + \left.\frac{1}{(4\pi F)^2}\sum_{i=1}^{24} b_i O_i \right) N_v \; .
\end{aligned}
\tag{4.10}
$$

Although quite a bit more involved, this Lagrangian has the same structure as (4.7). There is a piece with coefficients completely fixed in terms of LECs of $O(p)$ and $O(p^2)$. The second part has 24 new LECs b_i. The associated field monomials O_i can be found in Ecker and Mojžiš (1995c). It is this second part that is needed to absorb the divergences of the one–loop functional. The splitting of the b_i into divergent and finite parts introduces a scale dependence of the finite, measurable LECs $b_i^r(\mu)$. This scale dependence is governed by β–functions that are determined by the divergence functional (Ecker, 1994). Adding the finite part of the one–loop functional, one arrives at the total generating functional of Green functions in the pion–nucleon system to $O(p^3)$:

$$
Z = Z_1(g_A) + Z_2(a_i; g_A, m) + Z^{L=1}_{3,\text{finite}}(g_A;\mu) + Z_3^{\text{tree}}(b_i^r(\mu); a_i, g_A, m) \; . \tag{4.11}
$$

The functionals Z_1, Z_2 are tree–level functionals, whereas the functional of $O(p^3)$ consists of both a loop and a tree–level part. The sum of those two and therefore the complete functional is independent of the arbitrary scale μ.

The functional (4.11) contains the complete low–energy structure of the πN system to $O(p^3)$. In order to extract physical amplitudes from this functional, one has to calculate the appropriate one–loop amplitudes contained in $Z_3^{L=1}$. This has already been done for many processes of interest and I refer especially to the review of Bernard et al. (1995) for an extensive coverage of the phenomenological applications. Here, I want to consider an illustrative example of the class of amplitudes (or Green functions more generally) that are insensitive to the LECs b_i of $O(p^3)$. This class of amplitudes is of course of special interest for comparison with experiment because one does not need to know anything about the actual values of the renormalized LECs b_i^r. This is welcome because we are far from having very reliable information on most of these LECs.

This class of amplitudes can still be divided into two groups. In the first group, loop amplitudes do contribute, which are then necessarily finite because there are no available counterterms that could absorb the divergences. A well–known example is neutral pion photoproduction at threshold, $\gamma N \to \pi^0 N$, where it has been found only relatively recently (Bernard et al., 1991) that there is a sizeable loop contribution of $O(p^3)$. In this talk I want to discuss an example of the second class where there are neither loop nor counterterm contributions at $O(p^3)$. The only possible other contribution at this order must then come from the terms with fixed coefficients in (4.10).

As an example, consider nucleon Compton scattering at small photon energies in the forward direction. In a gauge where the polarization vectors have vanishing time components, the forward scattering amplitude has the form

$$
T = c_0\, \vec{\varepsilon}\,' \cdot \vec{\varepsilon} + i\, c_1\, \delta\, \vec{\sigma}\cdot(\vec{\varepsilon}\,'\times\vec{\varepsilon}) + O(\delta^2) \; , \tag{4.12}
$$

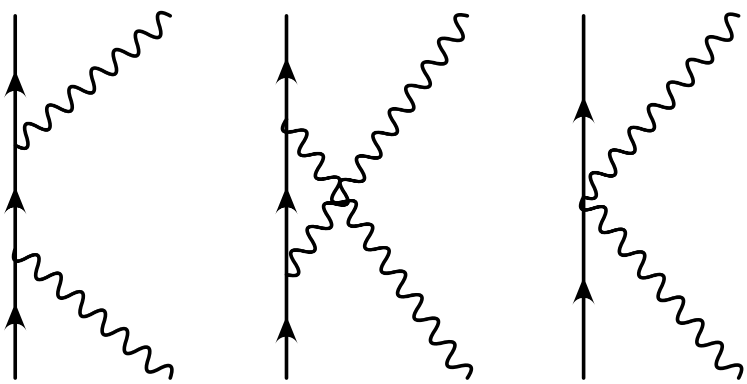

Figure 1: Tree diagrams for nucleon Compton scattering that are responsible for the leading coefficients c_0, c_1 of the forward scattering amplitude at low photon energies.

where $\delta = E_\gamma/m$. The coefficient c_0 determining the leading spin–independent amplitude of $O(p^2)$ is given by the Thomson limit

$$c_0 = -\frac{q_N e^2}{4\pi m} \tag{4.13}$$

with q_N the nucleon charge. As previously mentioned, this coefficient is directly given by the first term in the Lagrangian (4.7).

How does HBCHPT account for the leading spin–dependent Compton amplitude in terms of c_1? It is not very difficult to convince oneself that there is indeed neither a loop contribution nor a contribution proportional to the LECs b_i. The relevant diagrams are shown in Fig. 1 where the vertices in diagrams a,b are due to the Lagrangian (4.7), while the seagull vertex of diagram c comes from the $O(p^3)$ Lagrangian (4.10). From these Lagrangians one extracts the respective vertices (up to trivial factors)

$$\begin{aligned} k_2 &= a_6\tau_3 + \frac{a_7}{2} = \frac{1}{2}\begin{pmatrix} 1+\kappa_p & 0 \\ 0 & \kappa_n \end{pmatrix} \\ k_3 &= \frac{1}{2}(1+\tau_3)\left[\left(a_6 - \frac{1}{8}\right)\tau_3 + \frac{1}{2}\left(a_7 - \frac{1}{4}\right)\right] = \frac{1}{4}\begin{pmatrix} 1+2\kappa_p & 0 \\ 0 & 0 \end{pmatrix} \end{aligned} \tag{4.14}$$

in terms of the nucleon anomalous magnetic moments κ_N. The leading contribution to the spin–dependent Compton amplitude in the forward direction for small photon energies is of $O(p^3)$ and it is proportional to

$$k_2^2 - k_3 = \frac{1}{4}\begin{pmatrix} \kappa_p^2 & 0 \\ 0 & \kappa_n^2 \end{pmatrix} \tag{4.15}$$

in accordance with the classic low–energy theorem (Low, 1954; Gell-Mann and Goldberger, 1954)

$$c_1 = -\frac{\kappa_N^2 e^2}{8\pi m} \ . \tag{4.16}$$

5 CONCLUSIONS

CHPT is a systematic framework for analyzing the standard model at low energies. It has all the desirable features of a quantum field theory (unitarity, analyticity,...) even though it is non–renormalizable. The transition from the fundamental level of quarks and gluons to the effective level of hadrons generates a large number of effective coupling constants (LECs). Since these constants are not constrained by the symmetries of the standard model, additional phenomenological and/or theoretical input is needed to make CHPT predictive, especially in higher orders of the chiral expansion.

The chiral Lagrangian of the standard model is unique for the chosen number N_f of light flavours, but

- it has many different parts as shown in Table 1, and
- it can be organized in different ways (standard vs. generalized CHPT).

Elastic pion–pion scattering provides an excellent example that precise predictions are possible even to $O(p^6)$ (two–loop level) although there is a forbidding number of 111 coupling constants in the mesonic Lagrangian of $O(p^6)$. As chiral dimensional analysis suggests, the corrections of $O(p^6)$ are indeed small. Moreover, they are dominated by the unambiguous chiral logarithms whereas the contribution of the LECs of $O(p^6)$ is very small, especially for the S–waves. Therefore, pion–pion scattering is an almost ideal case for confronting QCD with forthcoming precision experiments at low energies. At the level of precision reached at $O(p^6)$, it is becoming necessary to include electromagnetic and isospin violating corrections.

In the pion–nucleon or more generally the meson–baryon system, we are still far from the precision attained in the meson sector. There are several obvious reasons for this difference: the baryons are not (pseudo–) Goldstone fields, resonances are in general much closer to the physical threshold, the chiral expansion has terms of all orders whereas only even orders appear in the meson sector,.... Nevertheless, there is considerable progress both on the more theoretical and on the phenomenological side also in this field. Heavy baryon CHPT provides a systematic low–energy expansion and the complete low–energy structure of the pion–nucleon system is now known to $O(p^3)$. However, many things remain to be done and the activity both in theory and experiment continues to grow.

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